Problem Set #4 – Due October 5, 1999

1) (From problem set #3) A 1971 Gallup poll asked Americans to rank them selves on a
    "happiness index" as follows: H = 0 (not happy), H = 1 (fairly happy), or H = 2 (very happy).
    Annual household income X (in thousands of dollars) was also recorded for each individual.
    The relative frequencies of various combinations of H and X were roughly as follows:

X	h		
	0	1	2
2.5	0.03	0.12	0.07
7.5	0.02	0.13	0.11
12.5	0.01	0.13	0.14
17.5	0.01	0.09	0.14

   ♣ Calculate the covariance and correlation.

2) The real return on an investment is its rate of increase corrected by the rate of inflation. You
    believe that the annual rate of return X on a portfolio of stocks will vary in the future with
    mean $\mu_X = 0.11$ and standard deviation $\sigma_X = 0.28$. Your further think that the annual real rate
    of return Y on Treasury bills will vary with mean $\mu_Y = 0.02$ and standard deviation $\sigma_Y = 0.05$.
    Assume that returns on stocks and Treasury bills vary independently.

    a. If you put half of your assets into stocks and half into Treasury bills, your overall return
        will be $Z = 0.5X + 0.5Y$. Calculate μ_Z and σ_Z.
    b. You decide that you will be willing to take more risk in exchange for higher return.
        Choose an allocation of your assets between stocks and Treasury bills that will
        accomplish this and calculate the mean and standard deviation of the overall return.
    c. Now assume that stocks and Treasury bills are no longer independent. Repeat the
        calculation of σ_Z in part a. assuming that the correlation between stocks and Treasury
        bills $\rho_{XY} = -0.1$.

3) In a desperate gamble, Wildcat Oil Exploration has committed all its remaining funds to
    finance a sequence of 12 drillings. Each drilling in this region has a 20% chance of
    successfully producing oil, independent of the other drillings.
    To avoid bankruptcy, three or more drillings must produce oil. What is the chance of this?
    (Hint: consult binomial probability tables in Wonnacott and Wonnaccott or use the
    BINOMDIST function in Excel).

4) If Z is a standard normal variable, calculate:

    a. P(Z > 1.60)   b. P(1.60 < Z < 2.30)   c. P(Z < 1.64)   d. P(-1.64 < Z < -1.02)
    e. P(0 < 1.96)   f. P(-1.96 < Z < 1.96)   g. P(-1.50 < Z < 0.67) h. P(Z < -2.50)

    (Hint: In this and the subsequent problems use table for normal distribution. If you want, you
    can also use the NORMSDIST function in Excel. Verify that you get the same results with
    both methods.)

5) Phil and Kim Bell don't know whether to buy a house or wait a year, in which case a price increase may put a house beyond their reach. Their best guess is that, if they wait a year, the price increase will be approximately normal, with a mean of 8% and a standard deviation of 10%.

   a. If the price increase exceeds 25% they feel they will be unable to afford the house. What is the chance of this?
   b. On the other hand, if the price drops, they will have won their gamble handsomely. What is the chance of this?

6) If X~N(10,3) and Y~N(15,8), and if X and Y are independent, what is the probability distribution of

   a. X + Y   b. X – Y   c. 3X   d. 4X + 5Y

7) Let X and Y represent rates of return (in percent) of two stocks. You are told that X~N(15,25) and Y~N(8,4), and that the correlation coefficient between the two rates of return is ρ_{XY} = -0.4. Now suppose you want to hold the two stocks in your portfolio in equal proportions. What is the probability distribution of the return on the portfolio? Is it better to hold this portfolio or to invest in only one of the two stocks? Why?

8) Honors option: Mercury Mufflers guarantees its mufflers for 3 years. On each muffler, they make a profit of $15. However, they must pay $50 for any replacement under the guarantee. (If this replacement also burns out within 3 years, they are *not* committed to a second replacement.)

   a. What is the average profit per sale (net, after paying for a possible replacement under the guarantee)?
   b. If they want this average profit per sale to be $5, by how much should they reduce the time period of the guarantee?