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Developing countries' borrowing in the international financial market with an uncertain credit ceiling[1]

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Abstract: The sovereign debt literature of the 1980s made the important point that lenders to a sovereign country will impose a credit ceiling on their lending, because at a sufficiently high level of debt the country will find it less onerous to default than to keep on servicing its debt. Provided that the borrower and its lenders agree on the level of the credit ceiling, there is no default in equilibrium. But is this a realistic assumption? If the credit ceiling is not clearly known or it changes widely with perceptions about the country's future prospects, so does the macroeconomic equilibrium of the borrowing country, making "debt problems" more likely. A baseline deterministic model shows that consumption, investment, and the current account deficit depend positively on the credit ceiling, implying that more optimistic expectations about the country's future access to external financing lead to larger current account deficits today. An extension of the model to the case where the credit ceiling is not known with certainty shows that the shadow cost of external credit exceeds its contractual interest rate. Therefore, optimal consumption, investment, and current account deficits are more moderate under uncertainty.

Key words: Capital movements, debt ceiling, financial crises, developing countries

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I. Introduction

After a decades long impasse that started with the Great Depression, private banks and institutional investors became once again important providers of finance to developing countries during the last quarter of the twentieth century. However, the access of developing countries to private sources of finance was far from smooth: periods of fast expansions of external liabilities were punctuated by external crises in which international banks and investors were reluctant to continue providing finance. The deepest and most protracted of these crises was the 1982 international debt crisis, which led to almost a decade of complicated rescheduling negotiations of international bank liabilities with a heavy involvement of the international financial institutions. Other crises followed in the 1990s: Mexico and Argentina in 1995, Thailand, Indonesia, and South Korea in 1997, Russia in 1998, Brazil in 1999, and Argentina in 2000. Although the recovery of the affected economies was much faster than after the international debt crisis of 1982, all these crises led to substantial recessions and increasingly larger rescue packages arranged by the international financial institutions.[1]

External financial crises are, needless to say, largely undesirable events. They affect the liquidity and return of financial investments in the affected countries and are onerous to the international financial institutions that provide emergency financing. More fundamentally, they are very costly to the countries affected by the crisis. Because of the sudden lack of external financing, they are forced to cut their domestic absorption abruptly, which normally leads to a recession through Keynesian multiplier effects. In addition, external crises are commonly associated with large currency devaluations which in the presence of dollar liabilities by domestic firms, banks, and the government leads to serious financial difficulties in the domestic economy.[2]

What is most puzzling about these external financial crises, given their dire effects, is that they have been preceded by periods of fast accumulation of external liabilities. With the benefit of hindsight it seems clear that these expansions were suboptimal ex-post, but why did countries choose to borrow so much in the first place? A conventional explanation is that the international financial investors' expectations about the country's prospects were fairly optimistic. These expectations, in turn, might have influenced the country's borrowing plans. As a Mexican central bank official once put it: "There seemed to be no limit on our supply of credit, no concern about our ability to service it… It seemed that nothing could go wrong" (Lissakers, 1991, p. 49). Krugman (1995) and Conley and Maloney (1995) point out that the implementation of market-oriented reforms has often led to overly optimistic expectations about a country's prospects.

Needless to say, developing countries can greatly benefit from access to external finance. Such finance can help the country speed up capital accumulation and economic growth. It can also allow a smoother intertemporal distribution of consumption expenses, making it possible to improve the standard of living of the population before the fruits of economic growth materialize. But in order to utilize external finance in an optimal way, the country should know what is its credit ceiling. The credit ceiling defines the shape of the country's intertemporal budget constraint. If the ceiling is not known with certainty, a country may make crucial mistakes in the intertemporal allocation of its consumption and investment expenses. In particular, the perception that the country is well considered by international investors may lead to excessive borrowing, which may make the country vulnerable to what Calvo (1998) termed a "sudden stop" (see also Calvo and Reinhart, 1999).

The baseline model presented in Section II is formally very similar to those of the sovereign debt literature of the 1980s. This literature made the important point that lenders to a sovereign country will impose a credit ceiling on their lending. The ceiling is based on the notion that at a sufficiently high level of debt the country will find it less onerous to default on its debt and endure a penalty imposed by the lenders than keep on servicing the debt (see e.g. Eaton, Gersovitz, and Stiglitz, 1986). In equilibrium default never occurs in these models because lenders enforce a credit ceiling that prevents countries from reaching such a critically high level of debt. In order to reach this equilibrium it is necessary to assume that lenders can agree in their assessment of what is the country's critical level of debt and co-ordinate their actions so the credit ceiling is enforced. This assumption is questionable: although the notion of a critically high level of debt must certainly be an important consideration in guiding lenders' actions, its precise level is never spelled out in international debt contracts. The problem is that countries need to know this precise level in order to formulate optimal borrowing plans. If the credit ceiling is unknown, the country must base its borrowing decisions, and hence its intertemporal allocation of expenses, on a guess.[3]

In the model presented in Section II, the country formulates optimal borrowing plans subject to an arbitrary credit ceiling. The credit ceiling represents the best available information about the country's access to international finance, but its determination is not considered in the model. The focus of the analysis is on the effect of the credit ceiling on the country's borrowing pattern. Will a higher credit ceiling lead to more borrowing and current account deficits? The model shows that this is indeed the case, implying that more optimistic expectations about the country's access to the international financial market lead to external disequilibria. Section III extends the model to the case in which the uncertainty about the location of the credit ceiling is explicitly recognized in the borrowing country. The main implication of this extension is that larger current account deficits are associated with a higher risk in case that the country hits the credit ceiling unexpectedly. As a result, the optimal paths of consumption, investment, and the current account deficit are smoother under uncertainty.

II. Optimal borrowing with an arbitrary credit ceiling

Even if there is no institutional mechanism by which the country and the international investors can stipulate and enforce a clearly defined ceiling to the country's external indebtedness, the country can still base its optimal plan on an educated guess about what that ceiling would be. The resulting plan will be optimal in a conditional sense, depending on how appropriate the educated guess turns out to be. The model presented in this section is formally similar to previous work in the sovereign debt literature (see e.g. Detragiache, 1992; Atkeson and Rios-Rull, 1996). A formal difference with the previous work is that the present model incorporates convex adjustment costs of investment, which make capital accumulate gradually rather than jump instantly to its steady state level. Yet, the most important difference with the previous literature is the interpretation of the debt ceiling as based on perceptions, which even if initially well-founded may turn out to be wrong ex post.

The economy is populated by n identical, infinitely lived families who, in each period, consume a composite good and supply one unit of labor. The preferences of each family for consumption over time are represented by

$$U = \int_0^\infty e^{-\theta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt, \quad \gamma > 0, \quad \gamma \neq 1, \tag{1}$$

where θ is the subjective rate of time preference and γ is the inverse of the intertemporal elasticity of substitution. The consumption good c_t is expressed in per capita terms.

The composite good is produced by a number of identical firms with a constant returns to scale technology that utilizes labor and capital as inputs. This technology can be represented by a strictly increasing and strictly concave function mapping production per capita on capital per capita, i.e. $y_t = f(k_t)$. I assume that $f(k_t)$ also satisfies $f'(0) = \infty$ and $f'(\infty) = 0$. The composite good can be used both for consumption and as capital. Capital accumulates according to

$$\frac{dk}{dt} = i_t - \delta k_t, \tag{2}$$

where i_t is gross investment per capita and δ is the rate of depreciation. Gross investment entails adjustment costs, represented by the quadratic function $g(i_t) = \frac{a}{2} i_t^2$, $a > 0$. Letting m_t denote net imports per capita, the resource constraint of this economy is given by

$$c_t = f(k_t) + m_t - i_t - \frac{a}{2} i_t^2 \,. \tag{3}$$

The country has a limited access to the international financial market. International investors are willing to buy a debt instrument issued by the country which pays a fixed interest rate r, but the total stock of debt they want to hold is limited by a credit ceiling. Denoting debt and the credit ceiling in per capita terms, respectively, by b_t and l, the evolution of the country's balance of payments is given by[4]

$$\frac{db}{dt} = rb_t + m_t \tag{4}$$

$$b_t \leq l \, . \tag{5}$$

Substituting from (3), the balance of payments equations can be written more succinctly as

$$\frac{db}{dt} = rb_t + c_t + i_t + \frac{a}{2} i_t^2 - f(k_t) \, . \tag{6}$$

The left hand side of this equation is the capital account surplus that is required to finance the current account deficit, given by the right hand side.

The problem of this economy is to decide at each point in time how much to consume, how much to invest, and how much to rely on external financing to achieve the first two purposes. To answer these questions assume the existence of a benevolent social planner wishing to maximize the welfare of the representative family. The social planner's decision problem consists in the maximization of (1) subject to (2), (5), and (6), given initial conditions for capital ($k_0 > 0$) and debt ($b_0 > 0$). The current-value Lagrangian of the problem is

$$L = \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_t [i_t - \delta k_t] + \mu_t [r b_t + c_t + i_t + \frac{a}{2} i_t^2 - f(k_t)] + \eta_t [l - b_t],$$

where the costate variables λ_t and μ_t and the Lagrange multiplier η_t represent, respectively, the shadow values of capital, external debt, and the foreign borrowing constraint. Equations (2) and (6) plus the following conditions are necessary and sufficient for a maximum:[5]

$$c_t^{-\gamma} = -\mu_t \tag{7}$$

$$\lambda_t = -\mu_t[1 + a\, i_t] \tag{8}$$

$$\frac{d\lambda}{dt} = \theta\,\lambda_t + \delta\,\lambda_t + \mu_t\, f'(k_t) \tag{9}$$

$$\frac{d\mu}{dt} = \theta\,\mu_t - r\,\mu_t + \eta_t \tag{10}$$

$$\lim_{t\to\infty} \quad e^{-\theta t}\,\lambda_t\, k_t \;=\; 0 \tag{11}$$

$$\lim_{t\to\infty} \quad e^{-\theta t}\,\mu_t\, b_t \;=\; 0 \tag{12}$$

$$l - b_t \geq 0 \qquad \eta_t \geq 0 \qquad \eta_t[l - b_t] = 0 \tag{13}$$

$$\mu_\tau^{+} - \mu_\tau^{-} \leq 0\,, \tag{14}$$

where τ is a junction time when the economy changes from unconstrained to constrained, or the other way around.

The dynamics of consumption and investment depend on whether the external debt is below or equal to the credit ceiling. While the external debt is below the credit ceiling the Lagrange multiplier η_t equals zero (by the complementary slackness condition 13), and consumption and investment behave as in the standard neo-classical model of an open economy (e.g. Blanchard and Fischer, 1989, pp. 58-69). While the external debt is equal to the credit ceiling the Lagrange multiplier is positive, and consumption and investment behave as in a closed economy with convex adjustment costs of investment. If the economy starts off unconstrained, it might continue unconstrained forever or it might become constrained, depending on the location of the credit ceiling. In this paper we are interested in the case where the economy experiences a transition from unconstrained to constrained in finite time. We start by analyzing the dynamic properties of this economy if it were forever unconstrained, then we will turn to the case in which the economy is forever constrained, and finally we consider the transition from the unconstrained to the constrained regime.

a. Unconstrained Economy

The path of investment for an unconstrained economy can be obtained by differentiating Equation (8) with respect to time and then substituting from Equations (8), (9), and (10) (taking into account that η_t=0 in the unconstrained regime, from Equation (13)):

$$\frac{di}{dt} = -\frac{1}{a}\left[f'(k_t) - (\delta + r)(1 + a\,i_t)\right]. \tag{15}$$

Although this equation does not have a closed-form solution, the dynamic path of investment can be studied qualitatively with the aid of a phase diagram. The dk/dt = 0 and di/dt = 0 loci are derived from Equations (2) and (15), respectively, as $i = \delta k$ and $i = \frac{1}{a}\left[\frac{f'(k) - \delta - r}{\delta + r}\right]$. At levels of investment above the dk/dt = 0 locus capital is increasing, while at levels of capital to the right of the downward sloping di/dt = 0 locus investment is increasing. Figure 1 shows the phase diagram, which has a downward sloping saddle point path: If the initial capital stock k_0 is below its steady state level k^0, capital increases and investment decreases along the optimal path.

Differentiating Equation (7) with respect to time and then substituting from Equations (7) and (10), the optimal path of consumption is

$$\frac{dc}{dt} = -\left[\frac{\theta - r}{\gamma}\right] c_t . \tag{16}$$

In order to rule out explosive paths of consumption I assume that $\theta \geq r$. After substituting into the balance of payments equation (6), Equation (16) integrates to

$$c_t = \left[r + \frac{\theta - r}{\gamma}\right]\left[\int_0^\infty e^{-rt}\left[f(k_t) - i_t - \frac{a}{2} i_t^2\right] dt - b_0\right] e^{-[(\theta - r)/\gamma]\, t}, \tag{17}$$

implying that consumption is either constant (if $\theta = r$) or decreases monotonically to zero (if $\theta > r$).

Given the optimal paths of investment and consumption, it follows that domestic absorption (the sum of consumption and investment) decreases over time. Because production increases as capital accumulates, the trade balance (domestic absorption minus production) must improve over time. In addition, it can be shown that the current account balance must be negative during the transition to the steady state[6] and zero at the steady state.[7] This path for the current account balance implies that the country's external debt increases monotonically until the steady state. If we let b^0 denote the steady state level of debt, and the credit ceiling is finite and satisfies $b^0 \leq l$, then the economy will be fully described by the unconstrained solution presented in this section. However, this solution will not feasible if $l < b^0$.

b. Constrained Economy

A monotonically increasing external debt may hit the credit ceiling before the steady state. If that happens, the economy will experience a transition to the constrained phase. As soon as the economy becomes constrained, the Lagrange multiplier η_t becomes positive (Equation 13) and the optimal paths of investment and consumption become:

$$\frac{di}{dt} = -\frac{1}{a}\left[f'(k_t) - (\delta + \rho_t)(1 + a\,i_t)\right] \qquad (18)$$

$$\frac{dc}{dt} = -\left[\frac{\theta - \rho_t}{\gamma}\right]c_t\,, \qquad (19)$$

where $\rho_t \equiv r - \eta_t/\mu_t$ represents the domestic interest rate. Recalling that μ_t is negative (Equation 7), it follows that the domestic interest rate of a constrained economy is greater than the international interest rate. The purpose of the higher domestic interest rate is to restrain consumption and investment expenditures so that the current account remains in equilibrium as the economy moves towards the steady state.

In order to study the dynamics of the constrained economy, first consider the case where $\theta = r$. Because $\rho_t > r$, it follows from Equation (19) that consumption has an increasing path. Recalling from our analysis of the unconstrained economy that the path of investment is decreasing when $\rho_t = r$, it is clear from Equation (18) that investment may either decrease over time, though at a smaller rate, or increase over time in the constrained economy. In sum, when the country does not have access to external borrowing, it cannot afford to increase its domestic absorption beyond its GDP. Therefore, the path of consumption is similar to the path of the

GDP, and capital accumulation is slower.

When $\theta > r$ it does not follow immediately from Equation (19) that the path of consumption is increasing. To prove the generality of this result, notice that the constrained economy is analogous to a closed economy whose problem is the maximization of Equation (1) subject to Equation (2) and the resource constraint

$$c_t = f(k_t) - rl - i_t - \frac{a}{2} i_t^2 . \tag{20}$$

The latter is obtained from Equation (6), by noticing that in the constrained economy $db/dt = 0$ and $b_t = l$. Except for the adjustment costs of investment, $\frac{a}{2} i_t^2$, this closed economy is identical to the to the standard infinite horizon neoclassical growth model (see, e.g., Blanchard and Fischer, pp. 38-47). Although the presence of adjustment costs makes the analysis more cumbersome, it can be shown that this economy has a saddle point equilibrium in which consumption increases monotonically as capital accumulates.[8] Therefore, it follows from Equation (19) that the domestic interest rate ρ_t exceeds θ during the transition to the steady state.

In the steady state the domestic interest rate ρ_t converges to θ. When $\theta = r$, the steady state levels of capital, investment, and production are identical in the constrained and unconstrained economies.[9] However, because l is less than the steady state level of debt of the unconstrained economy, it follows from (20) that steady state consumption is higher in the constrained economy. When $\theta > r$, the steady state levels of investment, capital, and production are lower in the constrained economy than in the unconstrained economy, but consumption is larger in the constrained economy because it converges to a positive level.

c. Optimal Regime Transition and Implications

If the economy starts off unconstrained but the credit ceiling is less than the steady state level of the unconstrained economy ($l < b^0$), the economy must experience a transition to the constrained regime. At the time τ when the external debt reaches the credit ceiling, the current account deficit has to be eliminated. The question is how this adjustment takes place. Equations (7) and (14) rule out a discrete increase of consumption at time τ. However, a discrete drop of consumption is not optimal either, because of the strict concavity of the utility function.[10] If consumption is continuous at the time τ, it follows that investment (from Equation 8) and the current account deficit (from Equation 6) should also be continuous at time τ.

Figure 2 illustrates the optimal paths of production, domestic absorption, and the current account balance of an economy that starts off unconstrained but reaches the credit ceiling at time τ. Before τ the economy is unconstrained; therefore, capital accumulates fast and production increases rapidly. At the same time, domestic absorption is initially very high and decreases over time. As a result, the economy starts with a very high current account deficit that is gradually corrected over time. At time τ the external debt reaches the credit ceiling and the economy becomes constrained. Between τ and the steady state capital accumulation and the growth of production slow down. Because the country cannot borrow abroad, it must now run a constant trade surplus to cover its debt services (*rl*), implying that the path of domestic absorption must track the increasing path of production during the constrained regime. Finally, notice that the adjustment of the current account at time τ is smooth: correcting the deficit that prevailed during the unconstrained regime does not require a discrete drop in domestic absorption.

A first implication of this model is that the optimal allocation of consumption and investment, and consequently of the current account deficit, depends on the perceived ceiling to the country's external debt. For example, if the credit ceiling is expected to be much tighter, the path of the current account shown in Figure 2 will not be optimal because the country would hit the credit ceiling much earlier than at time τ. The initial current account deficit in that case will need to be much smaller than in Figure 2, implying that consumption and investment will need to be much smaller.

A second implication of the model is that if the location of the credit ceiling turns out to be different than initially expected, the country's economic plan will be suboptimal ex-post. If the credit ceiling was perceived to be too tight, consumption and investment would be less than optimal, and the current account deficit would be corrected at a time when the country could potentially benefit from more access to international finance. If this happens, a new optimal plan could be devised later, based on the information that international investors are actually willing to have a larger exposure to the country's financial assets. A more serious error could occur if the optimal plan was based on over-optimistic expectations about the country's debt ceiling. In that case, consumption and investment would be too high, and the country would be forced to adjust its current account abruptly at the time its external debt reaches the ceiling.

Figure 3 illustrates three examples of paths that are suboptimal ex-post. The first one depicts a very prudent or conservative country, which formulates its optimal plans under the assumption that its access to the international financial market is rather limited. Because the country adjusts the current account deficit when it still has access to further borrowing, its plans are revised and a new cycle of borrowing takes place. These cycles continue until the country is actually unable to keep borrowing at time τ. The path labeled as "Overborrowing I" shows the case of an overoptimistic country, which assumes it will have a very generous future access to the international financial market. At time τ the country is still running a sizable current account deficit; therefore, the adjustment of the current account is not smooth, as the country is forced to cut domestic absorption abruptly. Finally, the path labeled "Overborrowing II" shows an overoptimistic country that goes through several revisions of its optimal plans. These revisions presumably take place as news about the country's access to the international financial market are updated. This path can describe the situation of a country that accesses the international financial market for the very first time. Initially, the country's borrowing is very prudent, but as it succeeds in obtaining financing, the country revises its expectations upwards, increasing its borrowing over time.

III. Uncertainty

Since access to the international financial market entails uncertainty and risks for the borrowing country, it makes sense to incorporate such features into the model. A simple way to do it is by assuming that the country faces two possible states of the world: unconstrained and constrained. In the unconstrained state, the country has access to the international financial market subject only to a solvency constraint. In other words, initially its foreign borrowing is only constrained by the standard no-Ponzi-game condition

$$\lim_{t\to\infty} \quad e^{-rt} b_t \;=\; 0\,. \tag{21}$$

In the constrained state, international investors do not want to increase their exposure to the country's financial assets. If the country is unconstrained at time *t*, there is a constant probability βdt that an unexpected financial crisis will take place between *t* and *t+dt*, in which case the country will become constrained. For simplicity, I assume that once the country looses its access to the international financial market, it remains constrained forever.

If the country is unconstrained at time *t*, the optimal levels of consumption and investment should satisfy the following Bellman equation

$$V(k_t, b_t) = \max_{c_t, i_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} dt + \frac{1}{1+\theta dt} \left[\left(1-\beta dt\right) V\left(k_{t+dt}, b_{t+dt}\right) + \beta dt\, V^C\left(k_{t+dt}; \bar{b}\right) \right] \right\} \tag{22}$$

subject to Equations (2), (6), and (21), where V^C is the value function of the deterministic constrained economy.[11] Multiplying both sides by $(1+\theta dt)$, dividing by *dt*, and letting $dt \to 0$, we can express the Bellman equation as[12]

$$\theta V(k_t, b_t) = \max_{c_t, i_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \frac{dV(k_t, b_t)}{dt} + \beta\left[V^C\left(k_t; \bar{b}\right) - V\left(k_t; b_t\right) \right] \right\} \tag{23}$$

$$= \max_{c_t, i_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + V_k\left[i_t - \delta k_t\right] + V_b\left[r b_t + c_t + i_t + \frac{a}{2} i_t^2 - f(k_t) \right] + \beta\left[V^C(k_t; \bar{b}) - V(k_t, b_t) \right] \right\},$$

where I use Equations (2) and (6) to expand the total derivative of *V* with respect to time in the second line, and V_k , V_b represent the partial derivatives of *V* with respect to capital and debt.

As I show in the Appendix, the optimal paths of consumption and investment are

$$\frac{dc}{dt} = -\left[\frac{\theta - r}{\gamma}\right]c_t + \frac{\beta}{\gamma}\left[\left(\frac{c_t}{c_t^C}\right)^{\gamma} - 1\right]c_t \tag{24}$$

$$\frac{di}{dt} = -\frac{1}{a}\left[f'(k_t) - (\delta + r)(1 + a\,i_t)\right] + \beta[i_t - i_t^C]\left(\frac{c_t}{c_t^C}\right)^{\gamma}, \tag{25}$$

where c_t^C and i_t^C denote the optimal initial levels of consumption and investment if the economy enters the constrained regime at time *t*. The law of motion of consumption can be written more succinctly as

$$\frac{dc}{dt} = -\left[\frac{\theta - (r + \sigma_t)}{\gamma}\right]c_t, \qquad \sigma_t \equiv \beta\left(\left[\frac{c_t}{c_t^C}\right]^{\gamma} - 1\right), \tag{26}$$

which is formally analogous to the optimal consumption paths of the unconstrained deterministic economy (Equation 16), with the only exception that the interest rate is augmented by the risk premium σ_t.

In order to characterize the solution, we can draw on our knowledge of the deterministic economy. For brevity I focus on the optimal path of consumption in the case where $\theta = \text{r}$. A first observation is that, at any point in time before the steady state, consumption should be bounded below by the optimal consumption level of the constrained deterministic economy: $c_t > c_t^C$. The reason is that the country can always afford to consume at least c_t^C, regardless of the conditions of the international financial market, and, as we have discussed in Section II, it can always do better by borrowing some money abroad. If $c_t > c_t^C$, the risk premium σ_t should be positive in Equation (26), implying that consumption grows over time, as in the constrained deterministic economy (see Equation 19).

A second observation is that, at any point in time before the steady state, consumption should be bounded above by the optimal consumption level of an unconstrained deterministic economy that starts with an external debt of b_t, which I denote by c_t^U.[13] Since consumption should grow over time (Equation 26), it cannot start at or above c_t^U. If it did, it would diverge from its steady state level, eventually violating the no-Ponzi-game condition (Equation 21). As a result of this and the previous observation, the optimal level of consumption must lie between c_t^C and c_t^U.

A final observation is that the risk premium should decrease over time, converging to zero in the steady state. This is a consequence of the result that consumption is bound by c_t^C and c_t^U. As the deterministic economy converges to the steady state, the gap between c_t^C and c_t^U converges to zero. As a result, the gap between c_t and c_t^C and, therefore, the risk premium σ_t also converge to zero.

Figure 4 illustrates this solution. At each point in time, c_t^C and c_t^U represent the optimal levels of consumption of a deterministic economy that starts, respectively, constrained and unconstrained at time *t*. From the argument above, the optimal solution for the stochastic economy, c_t, must lie between c_t^C and c_t^U. Notice that the optimal solution for the stochastic economy involves some external borrowing; as a result, c_t^U must decrease over time. The graph assumes that the economy reaches (or is very close) to the steady state at time t^0; at that time the paths of c_t^C, c_t^U, and c_t converge to the same level of consumption.

In order to complete the characterization of the solution notice that the optimal path of investment can be rewritten as

$$\frac{di}{dt} = -\frac{1}{a}\left[f'(k_t) - (\delta + r + \sigma_t)(1 + a\, i_t)\right], \qquad \sigma_t \equiv \beta a \frac{(i_t - i_t^C)}{\left(1 + a i_t\right)} \left[\frac{c_t}{c_t^C}\right]^\gamma . \tag{27}$$

As before, this expression is analogous to the optimal path of the unconstrained deterministic economy (Equation 15) with the addition of the risk premium σ_t to the international interest rate. The sum of the risk premium and the international interest rate represents the domestic interest rate of the stochastic economy. Because in equilibrium the interest rate should be the same for consumption and investment decisions, the two expressions for the risk premium presented in Equations (26) and (27) can be equated, yielding

$$\frac{c_t^\gamma - (c_t^C)^\gamma}{c_t^\gamma} = \frac{a(i_t - i_t^C)}{1 + a i_t} . \tag{28}$$

Given the argument above that $c_t > c_t^C$ in the stochastic economy, it must be also the case that $i_t > i_t^C$.

Finally, notice that in a deterministic economy that becomes constrained at time t the resource constraint is $f(k_t) = c_t^C + i_t^C + \frac{a}{2}(i_t^C)^2 + rb_t$ (see Equation 20). Then, it follows from Equation (6) that the current account deficit of the stochastic economy is

$$\frac{db}{dt} = (c_t - c_t^C) + (i_t - i_t^C) + \frac{a}{2}\left(i_t^2 - (i_t^C)^2\right). \tag{29}$$

Because the risk premium is an increasing function of (c_t - c_t^C) and (i_t - i_t^C), we can conclude that the risk premium is also an increasing function of the current account deficit. The intuition of this last result is straightforward. The larger the current account deficit, the larger the cuts in domestic absorption that would be required to balance the current account if capital inflows suddenly stop. To reduce this risk, the domestic interest rate must be higher in order to discourage consumption and investment.

IV. Concluding Remarks

The main message of this paper is that expectations about a country's access to international finance can by themselves influence the country's current expenditures and the size of its current account deficit. When a country perceives that international investors are optimistic about its prospects and willing to provide financing, it naturally wishes to take advantage of this financing both to accelerate capital accumulation and to smooth the intertempral distribution of consumption. The problem is that the precise credit ceiling it faces is not known. Countries, therefore, face the risk of borrowing too much and running current account deficits that are too high.

This paper has omitted modeling explicitly the process of expectation formation by international investors. Making the credit ceiling (in Section II) and the probability of a financial crisis (in Section III) exogenous is an important simplification of a much more complex reality, since country-specific factors are surely taken into account by international financial investors when deciding how much to invest. Yet, the recent literature on external crisis summarized in footnote 3 makes a compelling case for the importance of exogenous factors. Thus an exogenous credit ceiling not only simplifies substantially the analysis but also is in accordance with the recent literature.

The main policy implication of the analysis is the need to be prudent in the use of international finance. The model presented in Section III shows clearly that the presence of uncertainty about the country's future access to international finance should lead to moderation in expenditures and current account deficits. The more aware a country is about the possibility of suffering a sudden stop in its access to international finance, the more prudent it should be in its borrowing decisions. Prudential borrowing or, equivalently, running moderate current account deficits minimizes the adverse effects on the economy of a sudden stop.

Keeping fiscal deficits under control can help in order to achieve moderate current account deficits. However, fiscal prudence is not enough if the private sector follows expansionary spending policies. Therefore the objective of moderating external borrowing might require some policy instruments to check excessive private expenditures financed from abroad. In this context, for example, policies to control excessive external borrowing by domestic banks might be appropriate. Alternatively, a moderate tax on capital inflows may also help to moderate excessive private expenditures financed from abroad. Such tax could help bring the domestic interest rate closer to the shadow interest rate that results from the solution of the model of Section III, which exceeds the international interest rate.

An alternative implication of the analysis is that it could pay off to think about ways to ameliorate the lack of knowledge about a country's credit ceiling. Although the possibility of devising a single contract between a country and its international providers of finance that explicitly sets a credit ceiling is hard to imagine, it might be possible to improve on the current guessing game. Efforts under way to improve the speed and accuracy in the dissemination of information about borrowing countries will certainly help international investors formulate more realistic and better informed assessments of the countries' situation. Imperfect as they are, recently work on "early warning indicators" of financial crises is likely to help international financial institutions and market participants to better monitor the evolution of the borrowing countries and suggest corrective actions ahead of a crisis. Finally, novel financial instruments such as the contingent credit lines arranged by Argentina with international banks could also help to dispel the uncertainty about a country's future access to external finance. Hopefully, all these measures to improve transparency coupled with prudential expenditure policies by the borrowing countries aimed at minimizing the adverse effects of a sudden stop will contribute to ameliorate the severity of external financial crises in years to come.

Appendix

The first order conditions for a maximum on the right hand side of the Bellman equation are:

$$c_t^{-\gamma} + V_b = 0 \tag{A1}$$

$$V_k + V_b\left[1 + ai_t\right] = 0\,. \tag{A2}$$

To obtain V_b and V_k, differentiate the Bellman equation with respect to b and k using the envelope theorem

$$\theta V_b = V_{bb}\left[rb_t + c_t + i_t + \frac{a}{2}i_t^2 - f(k_t)\right] + rV_b + \beta\left[V_b^C - V_b\right] = \frac{dV_b}{dt} + rV_b + \beta\left[V_b^C - V_b\right] \tag{A3}$$

$$\theta V_k = V_{kk}\left[i_t - \delta k_t\right] - \delta V_k - V_b f'(k_t) + \beta\left[V_k^C - V_k\right] = \frac{dV_k}{dt} - \delta V_k - V_b f'(k_t) + \beta\left[V_k^C - V_k\right]. \tag{A4}$$

Once a shock is realized and the economy becomes constrained, the Bellman equation (22) reduces to

$$\theta V^C(k_t, \bar{b}) = \max_{c_t^C, i_t^C}\left\{\frac{\left(c_t^C\right)^{1-\gamma}}{1-\gamma} + V_k^C\left[i_t^C - \delta k_t\right] + V_b^C\left[rb_t + c_t + i_t + \frac{a}{2}i_t^2 - f(k_t)\right]\right\} \tag{A5}$$

subject to $b_t = \bar{b}$, where c_t^C and i_t^C are used to denote the optimal levels of consumption and investment of the constrained economy. The first order condition for a maximum are

$$\left(c_t^C\right)^{-\gamma} + V_b^C = 0 \tag{A6}$$

$$V_k^C + V_b^C\left[1 + ai_t^C\right] = 0 \tag{A7}$$

Equations (24) and (25) in the text are obtained after differentiating (A1) and (A2) with respect to time and substituting $\frac{dV_b}{dt}$ and $\frac{dV_k}{dt}$ from (A3) and (A4), V_b and V_k from (A1) and (A2), and V_b^C and V_k^C from (A6) and (A7).

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Figure 1: Unconstrained Economy - Phase Diagram of Investment and Capital

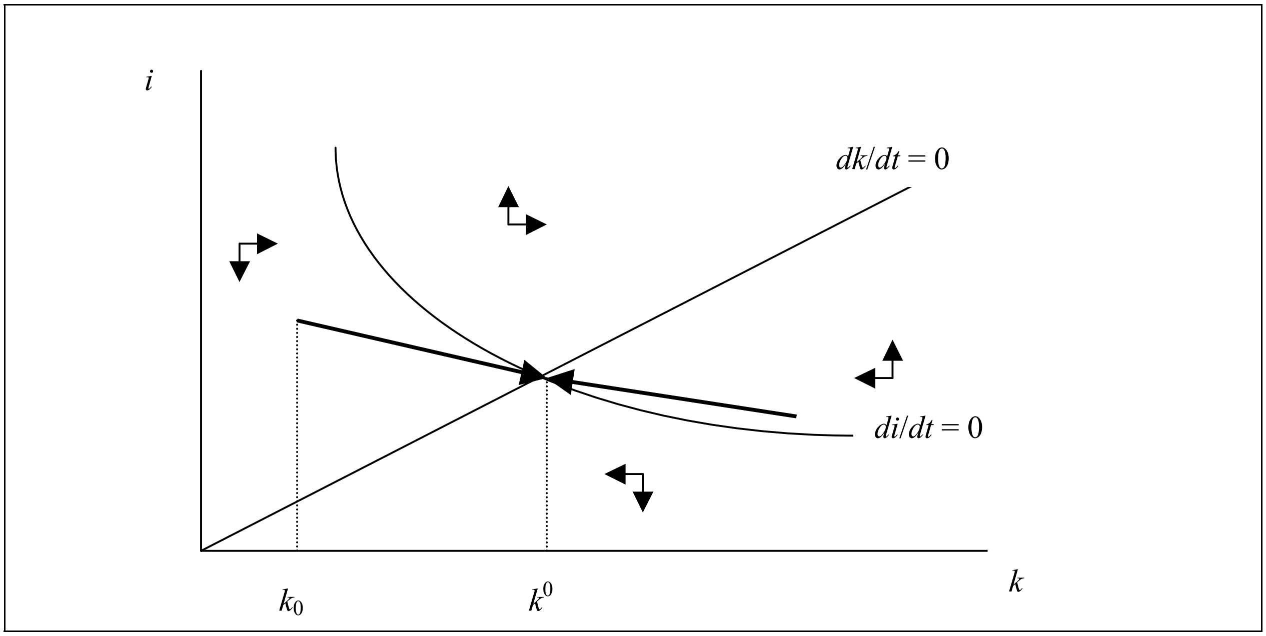


Figure 2: Optimal Paths of Domestic Absorption, Production, and the Current Account

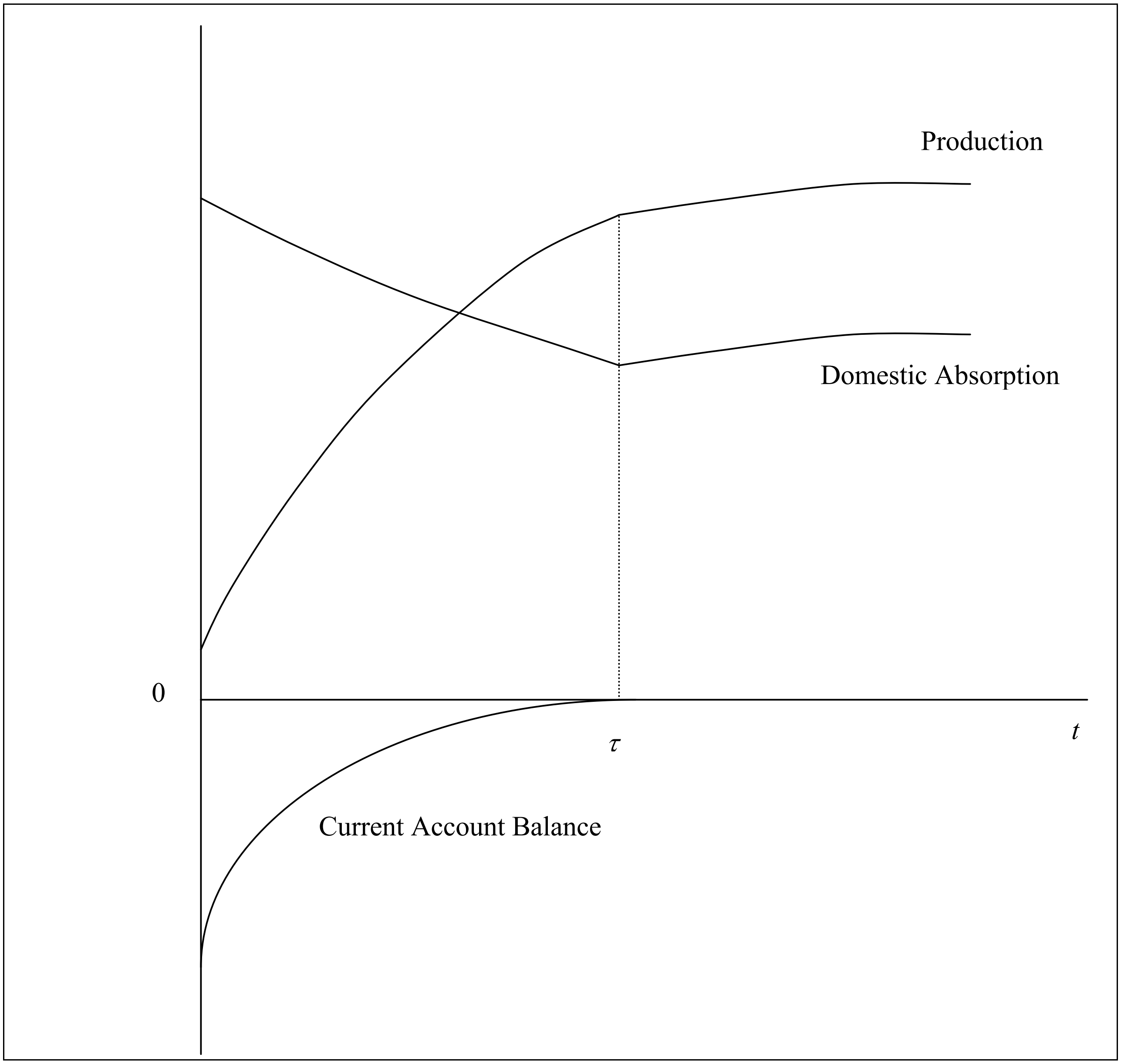


Figure 3: Suboptimal Paths of the Current Account

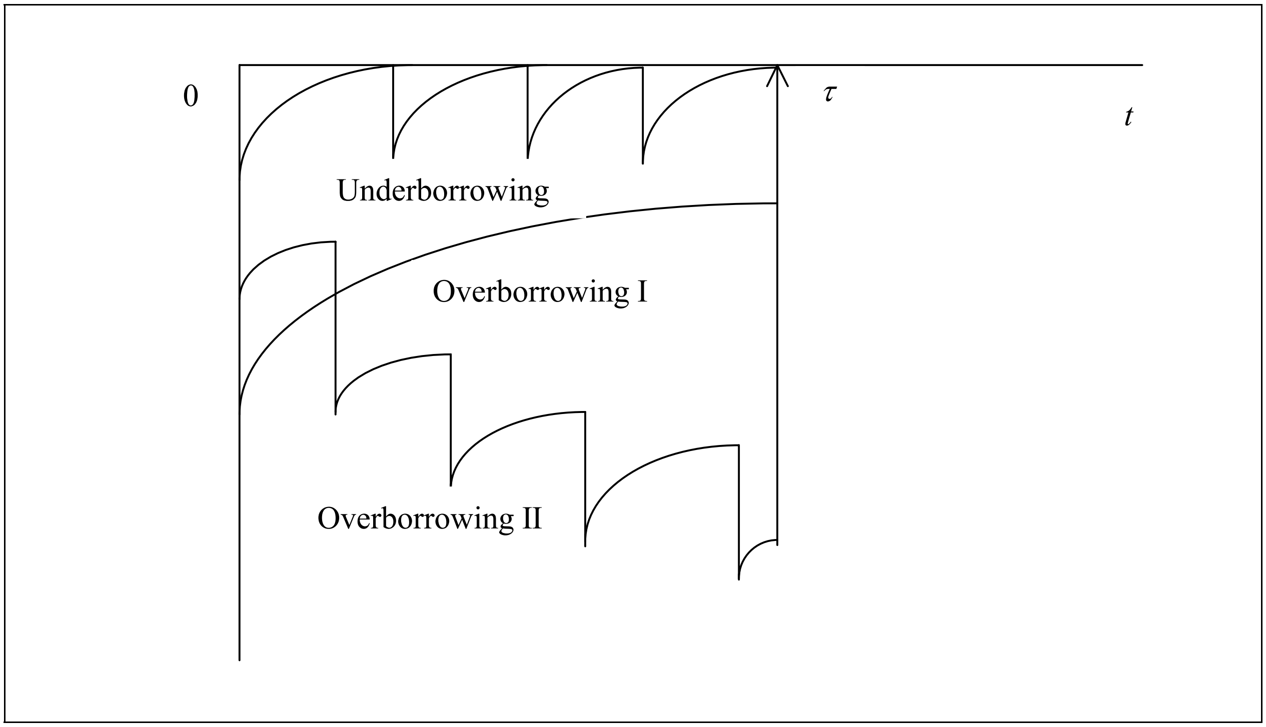
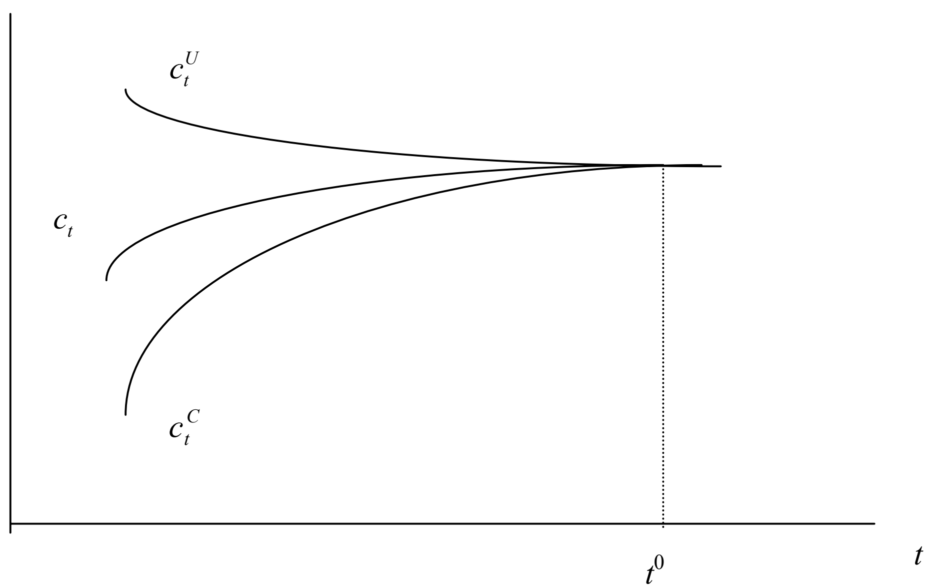


Figure 4: Optimal Consumption Path Under Uncertainty



Endnotes

[1] The increased frequency and magnitude of these crises has been a core motivation for the current discussions on the reform of the international financial system (see Fischer, 1999; Eichengreen, 1999; Rogoff, 1999).

[2] Countries that avoided devaluation by maintaining a strong peg to the dollar were not exempt from domestic financial problems. The 1995 crisis in Argentina led to a substantial reduction of peso deposits in the banking system, which also led to a domestic financial crisis.

[3] The recent literature has emphasized the difficulties of international investors in assessing the situation of individual borrowing countries. Aizenman and Marion (1999) mention the lack of timely information about countries' external debts. In addition, when not all market participants are well-informed, market perceptions of what is a safe level of exposure might be influenced by rumors or by events in other markets (Calvo, 1999). The possibilities of self-fulfilling prophecies, multiple equilibria, and contagion in emerging markets were subject of scrutiny after the Mexican crisis of 1995 (Calvo and Mendoza, 1996; Sachs, Tornell, and Velasco 1996). Finally, after the Asian crisis of 1997 new research further examined the possibility of financial panics and the institutional conditions under which borrowing countries are more vulnerable to them (Dooley, 1997; Chang and Velasco, 1998; Radelet and Sachs, 1998; Kaminsky and Reinhart, 1999; Rodrik and Velasco, 1999).

[4] For simplicity, the interest rate and the credit ceiling are assumed to be constant. The credit ceiling could be represented more generally as a function $l(r,z)$, where r is the interest rate and z is a vector of exogenous variables that consumers and firms use to forecast l. As long as z does not contain endogenous variables, there is no loss of generality in representing the credit ceiling as a constant.

[5] The first three terms of the Lagrangian correspond to the standard Hamiltonian. The last term adjoins the constraint to the level of the external debt (see Chiang, 1992, pp. 298-306).

[6] If the country had a current account surplus during the transition, then both the interest payments account and the trade balance would improve over time. As a result, the external debt would decrease to zero, and then international assets would increase without bound.

[7] If the country had a current account deficit in the steady state, then the external debt would increase without bound. Although this situation would not violate the transversality condition (12) when $\theta > r$, it would violate the requirement of solvency that lenders are likely to impose.

[8] Abel and Blanchard (1983) analyze a similar problem, with a slightly more complex adjustment cost function.

[9] To obtain the steady state levels of investment and capital, make *di*/*dt* = 0 and $\rho_t = r$ in Equation (18), and make *dk*/*dt* = 0 in Equation (2). The steady state level of production is obtained from the production function.

[10] Consider the level of consumption at two adjacent points in time: right before the transition $c_{\tau -}$ and at the transition $c_{\tau +}$. Because the utility function is strictly concave, it follows that $\left(u(c_{\tau-}) + u(c_{\tau+})\right)/2 < u\left((c_{\tau-} + c_{\tau+})/2\right)$ whenever $c_{\tau-} \neq c_{\tau+}$.

[11] V^C is defined as the maximum of (1) subject to (2), (6), and $b_t = \bar{b}$.

[12] See Dixit and Pindyck (1994, p. 113) or Whittle (1996, pp. 180-1) for a similar derivation.

[13] c_t^U is given by Equation (17) with b_0 replaced by b_t and the definite integral starting at time *t*.